Interpolation and unwrapping
Of sparse-grid InSAR data

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SUMMARY

• INTRODUCTION
• THEORY
• METHOD
• DATA
• RESULTS
• CONCLUSIONS
INTRODUCTION

• Phase Unwrapping (PU) reconstructs a phase function given its value modulo $2\pi$:

\[
\psi(x, y) = \angle(SLC_1 \cdot SLC_2^*) = \angle(INT) \cdot e^{i\phi(x, y)}
\]

\[
\psi(x, y) = W[\phi(x, y)] = \phi(x, y) + 2\pi \cdot K(x, y) \quad K(x, y) \in \mathbb{N} \quad \exists' -\pi \leq \psi(x, y) < \pi
\]

• PU algorithms have to be robust against aliasing (due to both steep variations of the original phase or noise).
INTRODUCTION

• New DInSAR techniques, such as Persistent Scatterers Interferometry (PSI), produce a sparse set of points which irregularly sample the deformation pattern. Such applications often require retrieval of regular-grid phase fields from sparse, wrapped data (e.g. atmospheric phase screen).
Sparse-grid phase interpolation

"Canonical" procedure

- Sampling → Phase Unwrapping on sparse grid → Interpolation

Unwrapping of sparse data:
- The "canonical" approach requires sparse data to be first unwrapped, then interpolated on a regular grid.
- Sparse-grid phase unwrapping algorithms exist and are under active development [Hooper & Zebker, Costantini...].
- However, they are influenced by the data sparsity --> critical sampling conditions:

γ = 0.8, 4674 samples
γ = 0.9, 760 samples
Our proposal:

• We empirically investigate the effect of interpolation of the complex phasor corresponding to the wrapped phase, bypassing the sparse PU step:

\[ z = \cos \phi + i \sin \phi; (|z| = 1) \]

• The solution is a raster, wrapped phase, which can be e.g.:
  ✓ used for phase corrections in PSI procedures (APS removal);
  ✓ unwrapped by robust and efficient raster algorithms such as Minimum-Cost Flow and similar – SNAPHU [Chen & Zebker, 2001].
A Similar approach was already proposed by Shanker and Zebker [“Sparse Two-Dimensional Phase Unwrapping Using Regular-Grid Methods”, *IEEE Geoscience and Remote Sensing Letters*, 6(2), 327–331, 2009.]

**Summary**

The sparse unwrapping problem is reduced to a regular 2D unwrapping problem using a nearest neighbor algorithm.

**Method**

1. Sparse phase data
2. Residues computed over the Delaunay triangulation of the data points $\hat{S}$
   - Interpolation Nearest Neighbor
   - The residues computed on $\hat{S}$ are equivalent to residues computed on the regular grid.
   - Regular-grid phase unwrapping can be used (SNAPHU)
Proposed method

• We use more sound interpolation functions and work on the complex signal.

• Working in simulation, the method is shown to give better results, in terms of both noise reduction and reconstruction performance, with respect to other approaches involving sparse-grid phase unwrapping.

• In particular we present results obtained by using Kriging interpolation, different regularity conditions of the original sampled surface, and different sampling densities.
Test procedures

1 – "Canonical" procedure

2 – Our procedure

3 – Shanker & Zebker procedure

\[ \psi = W(\phi) = W(\phi^0 + n) \]

\[ \psi^s \]

\[ \hat{\phi}^s \]

\[ \hat{\phi} \]

\[ \tilde{\psi} \]

\[ \tilde{\phi} \]

\[ \tilde{\phi} \]
Evaluation of results: RMSE

1) \( \sqrt{\frac{\sum W(w(\phi)-\psi)^2}{N}}; \sqrt{\frac{\sum W(w(\phi)-\psi)^2}{N}}; \sqrt{\frac{\sum W(w(\phi)-\psi)^2}{N}}; \) (re-wrapped absolute – wrapped original)

\[
\sqrt{\frac{\sum W(w(\phi)-w(\phi^0))^2}{N}}; \sqrt{\frac{\sum W(w(\phi)-w(\phi^0))^2}{N}}; \sqrt{\frac{\sum W(w(\phi)-w(\phi^0))^2}{N}};
\]
(absolute – original w/o noise)

2) \( \frac{\sum \phi - \phi}{N}; \frac{\sum \phi - \phi}{N}; \frac{\sum \phi - \phi}{N}; \) (absolute – original)

4) \( \frac{\sum (\phi - \phi^0)^2}{N}; \frac{\sum (\phi - \phi^0)^2}{N}; \frac{\sum (\phi - \phi^0)^2}{N}; \) (absolute – original w/o noise)

5) \( \frac{\sum (\phi^s - \phi^{0,s})^2}{N}; \frac{\sum (\phi^s - \phi^{0,s})^2}{N}; \frac{\sum (\phi^s - \phi^{0,s})^2}{N}; \) (absolute sampled – original sampled w/o noise)
Different levels of noise with Gaussian distribution are added to the surface.

Synthetic random surfaces with prescribed spatial autocovariance function can be generated, for example, through spectral filtering methods. We generated synthetic surfaces of size $5000 \times 5000$, obeying a Gaussian variogram (autocorrelation) model, with fixed parameters $N$ (Nugget) = 0, $S$ (Sill) = 3, $L$ (Range) = 1000, and from them, the simulated wrapped phase $\psi = \angle e^{i\phi}$. 
A random distribution of surface samples is extracted, using a spatial pattern coming from real X-band SAR data PSI processing. We analyze 3 different kinds of point patterns on the 3 areas, based on 3 different "coherence" thresholds (0.8, 0.85, 0.9). These masks are then placed on the simulate phase surface.

<table>
<thead>
<tr>
<th>AREA 1</th>
<th>AREA 2</th>
<th>AREA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \gamma_{th} = 0.80 ]</td>
<td>[ \gamma_{th} = 0.85 ]</td>
<td>[ \gamma_{th} = 0.90 ]</td>
</tr>
<tr>
<td>14257 samples</td>
<td>1830 samples</td>
<td>4674 samples</td>
</tr>
<tr>
<td>8119 samples</td>
<td>785 samples</td>
<td>2155 samples</td>
</tr>
<tr>
<td>4674 samples</td>
<td>261 samples</td>
<td>760 samples</td>
</tr>
</tbody>
</table>
Residues raster field of the original phase

<table>
<thead>
<tr>
<th>N. of Residues</th>
<th>AREA 1</th>
<th>AREA 2</th>
<th>AREA 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3689</td>
<td>3663</td>
<td>3832</td>
<td>49122</td>
</tr>
<tr>
<td>8385</td>
<td>8070</td>
<td>8471</td>
<td>49693</td>
</tr>
<tr>
<td>50057</td>
<td>49122</td>
<td>49693</td>
<td></td>
</tr>
</tbody>
</table>
Example of residues due to sparse samples

(*) = average density

Number of sparse-grid residues increases with noise level and point sampling sparsity
The phase residues are conserved in the nearest neighbor interpolated regular 2D image as Zebker has shown.

Instead, smooth interpolation (our method) decreases the phase residues.

The graphs show the numbers of residues in:
- Raster original surface
- Samples on sparse grid
- In the interpolated regular grid (kriging)
- In the interpolated regular grid (Nearest neighbor)
The phase residues are conserved in the nearest neighbor interpolated regular 2D image as Zebker has shown.

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- Samples on sparse grid
- In the interpolated regular grid (kriging)
- In the interpolated regular grid (Nearest neighbor)
Results of **wrapped phase**: RMSE

1) \[ \sqrt{\frac{\sum W(w(\phi) - \psi)^2}{N}}; \sqrt{\frac{\sum W(w(\bar{\phi}) - \psi)^2}{N}} \]

2) \[ \sqrt{\frac{\sum W(w(\phi) - W(\phi^0))^2}{N}}; \sqrt{\frac{\sum W(w(\bar{\phi}) - W(\phi^0))^2}{N}} \]

(both with noise)

(both without noise)

2 – Our procedure

1. Simulated phase + noise
2. Sampling
3. Interpolation (real and imaginary part)
4. Phase Unwrapping on regular grid

\[ \psi = W(\phi) = W(\phi^0 + n) \]

\[ \tilde{\psi} \rightarrow \hat{\psi} \rightarrow \psi \]
RMSE (Root Mean Square Error) is calculated between the simulate original wrapped phase plus noise and the restructured wrapped phase (1. ‘canonical’ one, 2. procedure implemented by us).

We investigate as the interpolation of complex signal gives results comparable to ‘canonical’ approach in which the PU is used.
The trend of RMSE is the same of the analyzed cases previously if we change the average density of samples at a fixed noise.

**INTRODUCTION**

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**CONCLUSIONS**
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RMSE (Root Mean Square Error) is calculated between the simulate original wrapped phase and the reconstructed wrapped phase (1. ‘canonical’ one, 2. procedure implemented by us).

<table>
<thead>
<tr>
<th>Std noise</th>
<th>RMSE 1st proc.</th>
<th>RMSE 2nd proc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.0676</td>
<td>0.0677</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2412</td>
<td>0.2411</td>
</tr>
<tr>
<td>0.9</td>
<td>0.2772</td>
<td>0.2714</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5699</td>
<td>0.4799</td>
</tr>
</tbody>
</table>

- **INTRODUCTION**
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- **RESULTS**
- **CONCLUSIONS**
Results of **absolute phase**: RMSE

1) \( \sqrt{\frac{\sum (\phi - \phi)^2}{N}}; \sqrt{\frac{\sum (\phi - \phi)^2}{N}}; \sqrt{\frac{\sum (\phi - \phi)^2}{N}} \) (with noise)

2) \( \sqrt{\frac{\sum (\phi - \phi^0)^2}{N}}; \sqrt{\frac{\sum (\phi - \phi^0)^2}{N}}; \sqrt{\frac{\sum (\phi - \phi^0)^2}{N}} \) (without noise)

**2 – Our procedure**

\[ \Psi = W(\phi) = W(\phi^0 + n) \]

\[ \tilde{\Psi} \]

\[ \tilde{\phi} \rightarrow \phi \]
RMSE (Root Mean Square Error) is calculated between the simulate original **absolute** phase plus noise and the reconstructed phase (1. ‘canonical’ one, 2. procedure implemented by us).

Even if in the our approach we use an PU algorithm on regular grid (as SNAPHU) to reconstruct the absolute phase, our results are better than ‘canonical’ ones.
The trend of RMSE is the same of the analyzed cases previously if we change the average density of samples at a fixed noise.
RMSE (Root Mean Square Error) is calculated between the simulate original **absolute** phase and the reconstructed phase (1. ‘canonical’ one, 2. procedure implemented by us)
Results of phase samples: RMSE

\[ 1) \sqrt{\frac{\sum(\phi^s - \phi^0)^2}{N}}; \sqrt{\frac{\sum(\bar{\phi}^s - \phi^0)^2}{N}}; \sqrt{\frac{\sum(\bar{\phi}^s - \phi^0)^2}{N}} \] (without noise)

2 – Our procedure

- Simulated phase + noise
- Sampling
- Interpolation (real and imaginary part)
- Phase Unwrapping on regular grid

\[ \psi = W(\phi) = W(\phi^0 + n) \]

\[ \vec{\psi} \]

\[ \vec{\psi}^s - \bar{\phi}^s \]
RMSE (Root Mean Square Error) is calculated between the simulate original absolute phase of samples and the reconstructed absolute phase for three procedures (1. canonical one, 2. procedure implemented by us, 3. Zebker’s one).
CONCLUSIONS

• Interpolation of sparse phase data requires previous phase unwrapping (PU)
• Sparse sampling usually worsens aliasing conditions, increasing chances of errors in PU results
• We proposed the use of smooth interpolation algorithms to retrieve a continuous phase field.
• Our procedure gives performance results slightly better or at least comparable to the "traditional" ones, in terms of both closeness to the original wrapped phase, and to the "ideal", absolute phase reconstruction.
• These performances add to the advantage of avoiding sparse PU (which is a complex and error-prone operation)
• Possible applications include e.g. the retrieval of smooth phase fields such as in the case of atmospheric phase contributions in PSI techniques, over the regular image grid starting from the PSI samples.